Truncated trapezoid velocity profiles

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March 12, 2010

We were recently working with the WAM robot and needed to generate trajectories from a haptic device. The WAM code-base had code for following a simple trapezoidal trajectory (see Fig. 1).

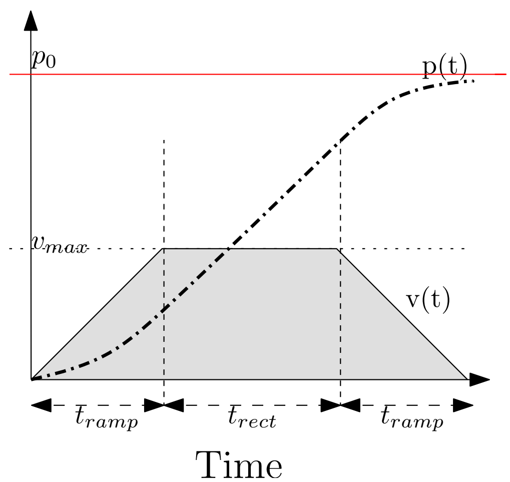

Figure 1: Trapezoid velocity profile to get from 0 to p_0 gives the position curve $p(t)$

It is straightforward to generate such a trajectory given maximum velocity and acceleration constraints (v_{max} and a_{max} respectively). The velocity in the ramp-up phase is, $v(t) = a_{max}t$, meaning the time to reach maximum velocity is $t_{ramp} = v_{max}/a_{max}$ and the distance travelled in this section is $\frac{1}{2}a_{max}t_{ramp}^2$. If If the change in position δp is greater than $a_{max}t_{ramp}^2$ then the curve must contain a period of constant max velocity:

$$t_{rect} = (\delta p - a_{max}t_{ramp}^2)/a_{max}$$

If δp is less than or equal to $a_{max}t_{ramp}^2$, then their is an equal period of time, t_{mid}, with ramp up and the ramp down:

$$\int_0^{2t_{mid}} v(t)dt = a_{max}t_{mid}^2$$

implying

$$t_{mid} = \sqrt{\frac{|\delta p|}{a_{max}}}$$

.

The WAM library routines allowed to issue more positional commands, but the previous trajectory would be halted and the new one started with an initial velocity of zero (more or less using the method above). Figure 2 illustrates the case when a command p_0 is issued at time 0, and then some time later a similar command p_1 is issued. Clearly the $p(t)$ curve is not smooth at $t_{interrupt}$ ($v(t)$ is discontinuous); this is evident in the robot motion. A worse case occurs when the commands are issued more frequently, implying that the trajectory can never reach full velocity.

Instead of creating a new trajectory with zero velocity, especially when the subsequent commands (e.g., p_1) are the same as the previous command, the trajectory should take into account the current velocity. So instead of the result in Figure 2, the interruption should still produce a curve more like that in 1. Regardless of where the interrupted commadn is received (or its location), the resulting positional trajectory $p(t)$ should obey the constriants a_{max} and v_{max}. This is illustrated in Figure 2).

The implementation is straightfoward. There are main two cases to handle. Like before, if the distance to travel is large, the trajectory will have a period of where the velocity is at v_{max}. There are

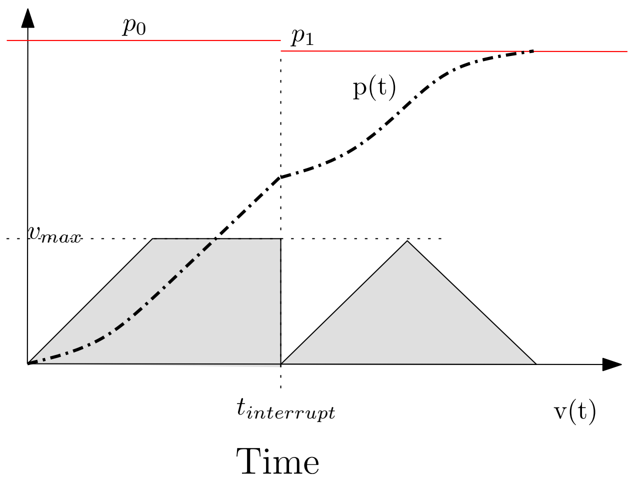

Figure 2: Initial command to go to p_0 is interrupted at $t_{interrupt}$. The WAM library would always create a new trajectory starting at zero velocity.

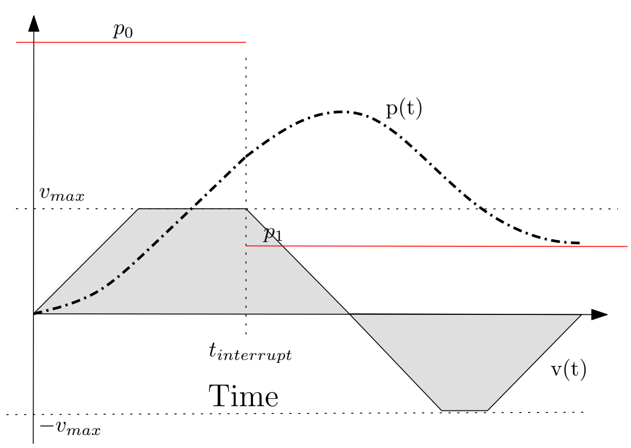

Figure 3: The created trajectory goes to the interrupted position but maintains the limits on the velocity and acceleration

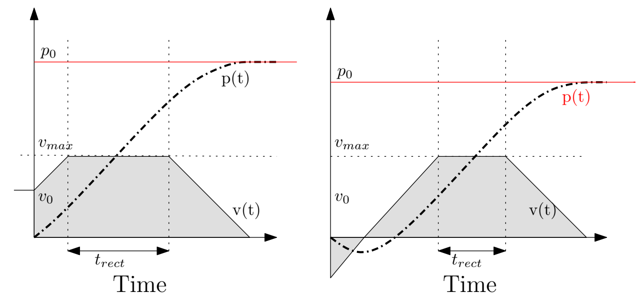

Figure 4: A period of constant velocity is needed to reach the new position. Notice that $p'(0) \neq 0$

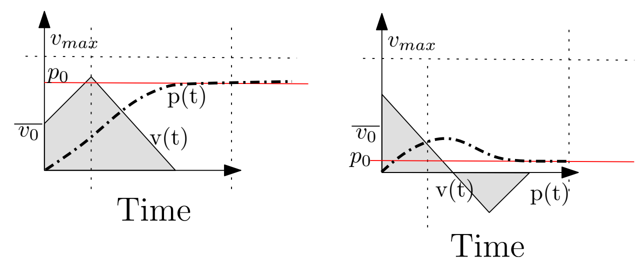

Figure 5: No constant velocity needed. The latter is a special case that occurs when the velocity is moving in the correct direction of δp but even if the curve were to decelarate to zero it would still overshoot the desired location. It then needs to have some negative velocity to compensate.

again a period of ramp-up and ramp-down; in this case, however, the ramp-up time may vary depending on the velocity of the current trajectory. It is always assumed that the robot should have zero velocity when it reaches the desired position. The time to ramp-up is, $t_{ramp\uparrow} = (v_{max} - v_0)/a_{max}$ implying that the distance traveled during ramp-up is $v_0 t_{ramp\uparrow} + \frac{1}{2} a_{max} t^2_{ramp\uparrow}$. The ramp down time is as it was in the simple case $t_{ramp\downarrow} = v_{max}/a_{max}$ giving a distance of $\frac{1}{2} a_{max} t^2_{ramp}$. Therefore when $\delta p > v_0 t_{ramp\uparrow} + \frac{1}{2} a_{max} t^2_{ramp\uparrow} + \frac{1}{2} a_{max} t^2_{ramp\downarrow}$ the velocity profile will have a period of constant velocity. Figure 4 illustrates this case.

The second main case is when the period of constant velocity is not needed (Figure 5). The solution can easily be obtained. Integrating the components of the curve, the total distance travelled as a function of the change of velocity $d(x) = a_{max}x^2 + 2v_{max}x + v_{max}^2/(2a_{max})$. Solving the quadratic equation $d(x) - \delta p = 0$ gives two solutions:

$$D = 4v_0^2 - 4a_{max}(v_0^2/(2a_{max}) - \delta p) \tag{1}$$

$$x_1 = (-2v_0 + \sqrt{D})/(2a_{max}) \tag{2}$$

$$x_2 = (-2v_0 - \sqrt{D})/(2a_{max}) \tag{3}$$

$$\tag{4}$$

In the case of positive acceleration, the second is always negative, so the trajectory has a period of positive acceleration followed by a period of negative acceleration that takes the velocity to zero and $p(t) = p_0$. Again, in the case of positive acceleration, $x_1 \leq 0 \Rightarrow (-2v_0 + sqrt(D)) \leq 0$ whenever

$$\sqrt{D} \leq 2v_0 \tag{5}$$

$$\Rightarrow D < 4v_0^2 \tag{6}$$

$$\Rightarrow 4a_{max}(v_0^2/(2a_{max}) - \delta p) \geq 0 \tag{7}$$

$$\Rightarrow v_0^2/(2a_{max}) \geq \delta p \tag{8}$$

The quantity, $v_0^2/(2a_{max})$, is exactly equal to the distance travelled if the trajectory starts at v_0 and decelrates to zero. For example,

$$v(t) = (v_0 - a_{max}t) \Rightarrow t_{zero} = v_0/a_{max}$$

and

$$\int_0^{tzero} v(t)dt = v_0 t - a_{max}t^2/2|_0^{t_{zero}} \tag{9}$$

$$= v_0 t_{zero} - a_{max}t_{zero}^2/2 \tag{10}$$

$$= v_0^2/a_{max} - a_{max}v_0^2/(2a_{max}^2) \tag{11}$$

$$= v_0^2/(2a_{max}) \tag{12}$$

Implying that the special case occurs when the distance to zero velocity is greater than δp. This special case (illustrated in Figure 5) occurs when the robot is currently moving in the same direction as the new desired pose, but even if the robot were to decelerate to zero velocity it would overshoot the desired pose. In this case, we solve first for the part of the curve where v_0 is crossed, and then append a triangular period of negative velocity. As this second part of the curve starts at zero velocity and ends at zero velocity it is the same as the simple trapezoid trajectory (e.g., solve for t_{mid}).

1 Results

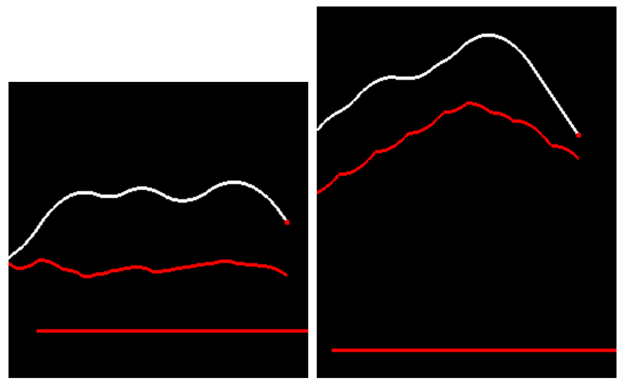

Figure 6: Sample trajectories generated using the improved trapezoid method (white) and the original one that always starts at zero velocity (red).

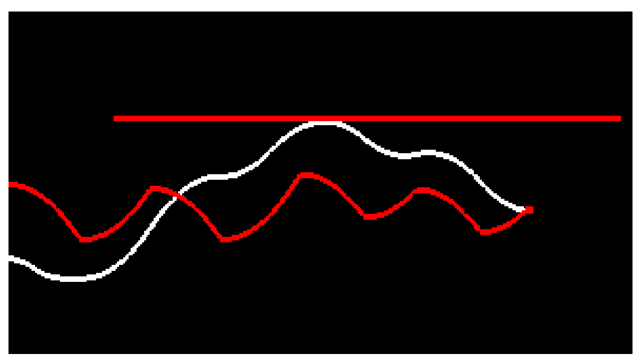

Figure 7: Another example

See Figure 6 for sample trajectories. Notice that the white curve is always smooth and always gets close to the desired location. The red curve always interrupts with a new trajectory at constant velocity. The right of the figure illustrates a case similar to Figure 2.