Shape-and-user-guided segmentation

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Motivation

Chan-Vese Segmentation
- Original functional
- Fixed shape (Chan-Vese like functional)
- Augmented functional

Optimization

Additional Constraints

Results/Demo
Motivation

- Segmentation with strong intensity prior and shape prior
  - known range of intensities for liver
  - shape similar between livers
- May need user-interaction during segmentation
Chan-Vese functional

\[ F(\phi) = \int_{\Omega} \left( -\log P_{in} H(\phi) - \log P_{out} (1 - H(\phi)) + \delta(\phi) |\nabla \phi| \right) \, dA \]

- **Shape prior:** regularized with curve length
Fixed Shape Energy

\[ F(\bar{p}) = \int_{\Omega} \left( -\log P_{in} \hat{H}(\bar{x}) - \log P_{out}(1 - \hat{H}(\bar{x})) + \delta(\phi)|\nabla \phi| \right) dA \]

\[ \hat{H} = H(\hat{\phi}(W(\bar{p}, \bar{x}))) \]

- single fixed prior shape (a levelset \( \hat{\phi} \))
- transformation, parameterized by \( \bar{p} \).
- curve length needed (e.g., when parameterization non-rigid)
Example transforms

Transforms

- rigid $\bar{p} = [\theta, t_x, t_y]$
- affine
- FFD
- ...

Template $W(p)$
Basic transforms limitations

- single transform not expressive enough
- potential solutions:
  - train FFD deformations to samples; use PCA on deformation parameters
  - do PCA on signed distance function on training data directly
  - allow arbitrary shape deformation (in levelset) but constrain it to be close to training data
Allowing deviation from a mean shape

Augmented cost functional

\[ F(\phi, \bar{p}) = \int_{\Omega} -\log P_{in} H(\phi) - \ldots + \lambda_{prior} F_{prior}(\phi, \bar{\phi}, \bar{p}) \]

- prior penalizes deviations from transformed mean (or training shapes)
- \( \lambda_{prior} \) tunes between no deviation to no prior.
Prior

\[ F_{\text{prior}} = \begin{cases} 
0 & \text{if } H(\phi(x)) = H(\bar{\phi}(W(\bar{p}, x))) \\
|\bar{\phi}(W(\bar{p}, x))| & \text{if } |\bar{\phi}(W(\bar{p}, x))| < \sigma \\
\infty & \text{otherwise}
\end{cases} \]
Prior

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Optimization

Joint optimization of segmentation and transformation parameters

\[ F(\bar{p}) = \min_{\theta} \int_{\Omega} -\log P_{in}H(\phi) - \ldots + \lambda_{\text{prior}} F_{\text{prior}}(\phi, \bar{\phi}, \bar{p}) \]

- Use conjugate gradient over parameters \( \bar{p} \), where cost functional is then evaluated optimally using graph-cuts
- Bandwidth sigma means problem only needs to be solved within band
  - Everything that maps to \( |\bar{\phi}(W(\bar{p}, x))| \geq \sigma \) is fixed, just need to adjust border weights appropriately.
Graph example

Assume bandwidth is 1.

- Green circle denotes transformed mean/template
- Magenta squares are always outside
- Green squares are inside.

![Graph example](image-url)
Selection indicates *mean shape* optimization uses this shape.
Example 1

Varying the hardness parameter.
Point Constraints

Point constraints can easily be embedded in the cost:

\[ E_{pt}(x_i) = \bar{\phi}(W(p, x_i)) \]
Sample Point Constraints
Demo